DOCUMENT RESUME

ED 440 880 SE 063 575

AUTHOR Chuan, Jen-chung
TITLE Geometric Constructions with the Computer.
PUB DATE 1995-12-00
NOTE 23p.; Lecture given at the ATCM (1st, Singapore, December 18-21, 1995).
PUB TYPE Guides - Classroom - Teacher (052) -- Speeches/Meeting Papers (150)
EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Computer Uses in Education; Elementary Secondary Education; *Geometric Constructions; Mathematics Activities; *Mathematics Instruction

ABSTRACT

The computer can be used as a tool to represent and communicate geometric knowledge. With the appropriate software, a geometric diagram can be manipulated through a series of animation that offers more than one particular snapshot as shown in a traditional mathematical text. Geometric constructions with the computer enable the learner to see and understand a diagram in different ways. Engaging in the construction of the animation encourages the learner to go through the abstract process of formulation of conjectures, generalization, condition-simplification, and classification. This paper offers examples of such constructions on topics such as enveloping curves, linkage, polynomial interpolation, inversion, hypocycloid, and epicycloid. (ASK)

Reproductions supplied by EDRS are the best that can be made
from the original document.

ED 440 880

Geometric Constructions with the Computer

Lecture given at the First ATCM, December 18-21, 1995, Singapore

Jen-chung Chuan

Department of Mathematics

National Tsing Hua University

Hsinchu, Taiwan 300

jcchuan@math.nthu.edu.tw

1 Introduction

Computer can be used as a tool to represent and communicate geometric knowledge. With the appropriate software, the geometric diagram can be manipulated through series of animation that offer more than one particular snapshot as shown in a traditional mathematical text. Geometric constructions with the computer enables the learner to see and understand a diagram in different ways, Engaging in constructing the animation encourages the learner to go through the abstract process of formulation of conjectures, generalization, condition-simplification and classification. In what follows we shall offer the visual experience of geometric animation by guiding the audience through demonstrations of geometric construction from topics in enveloping curves, linkage, polynomial interpolation, inversion, hypocycloid and epicycloid.

2 Envelop

When a system of lines or curves expressed in the form of an algebraic equation, it is not clear immediately if there is another curve touching every member of the system. A precise drawing allows the learner to perceive this notion of enveloping curve visually. Interesting enveloping curves occur when the system obeys certain common geometric conditions. It is thus desirable to generate the curve by means of the direct geometric notion. Consequently a dynamic geometric environment is particularly suitable for such exploration.

An enveloping curve may appear when only straight line segments are drawn:

SE 043 575

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY

J. Chuan

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

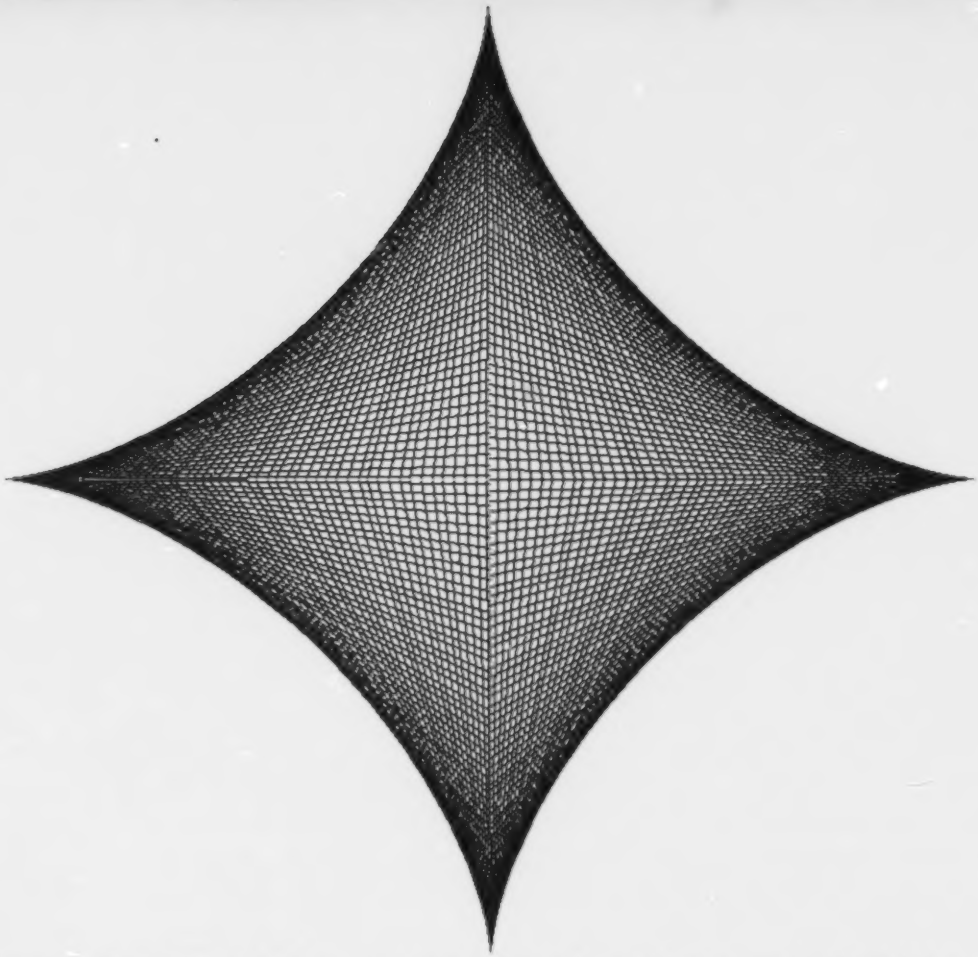
U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☒ This document has been reproduced as received from the person or organization originating it.

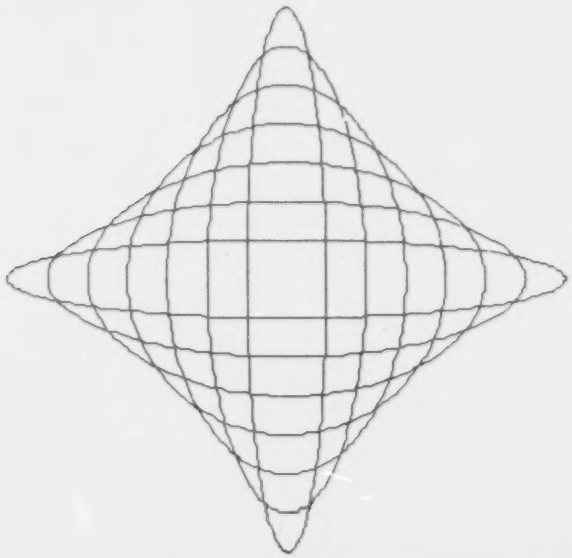
☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

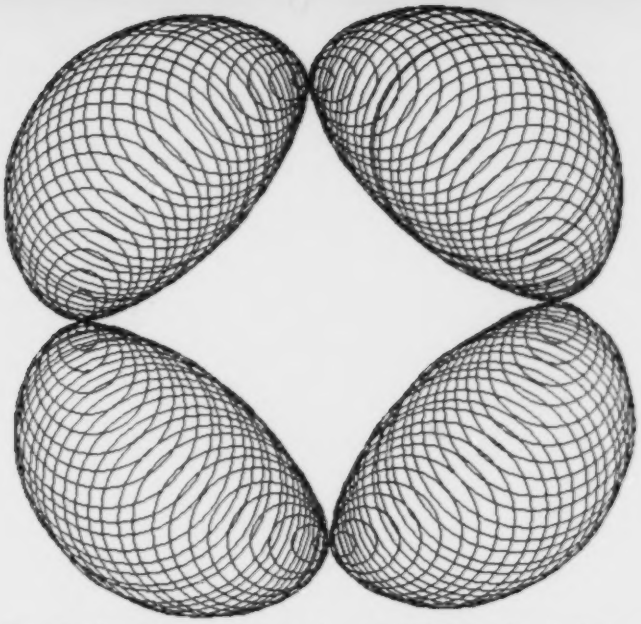
BEST COPY AVAILABLE



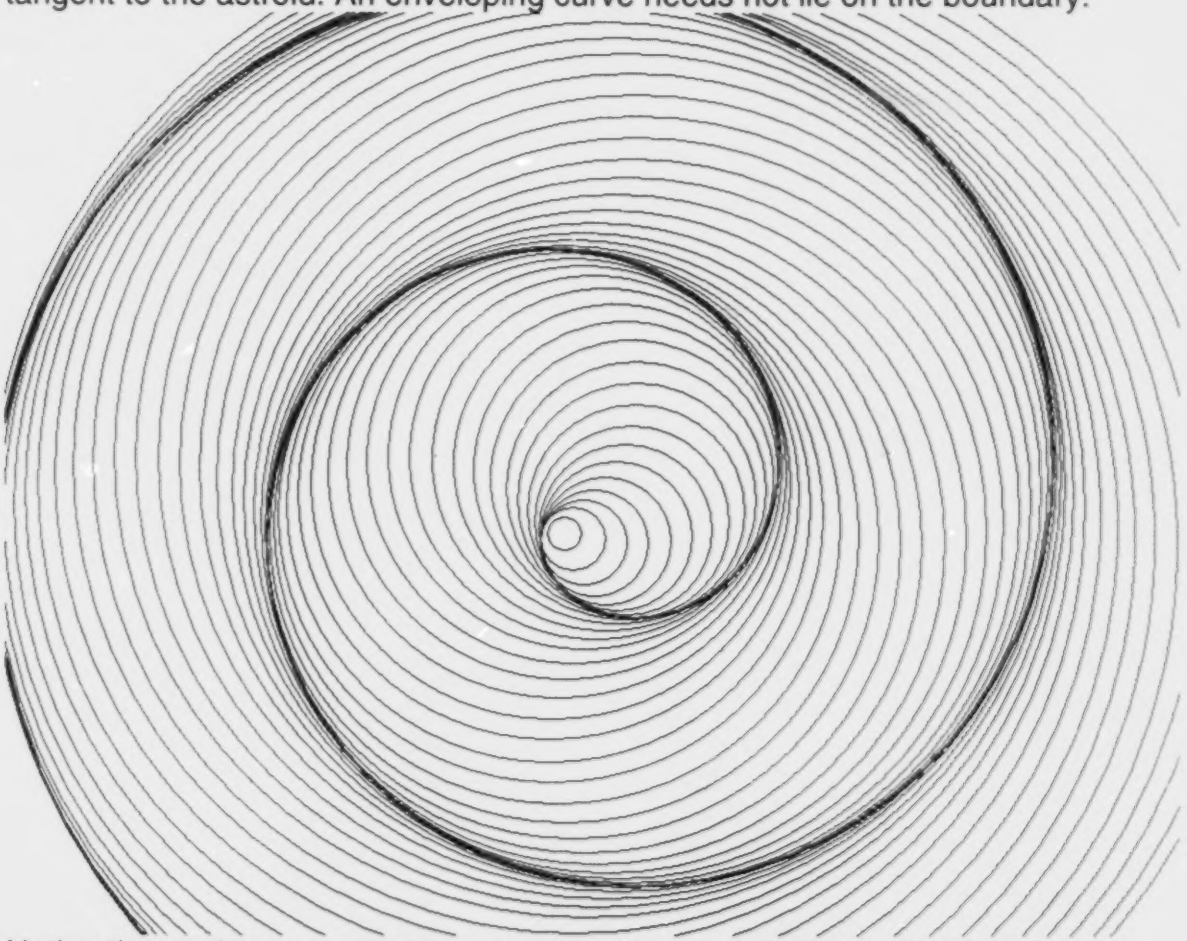
In this case all segments have the same length and have endpoints lying on two mutually perpendicular lines. The same envelop may appear as a curve touching a family of ellipses:



All ellipses have the same major axis, the same minor axis, and the same sum of distances from the foci. This astroid also appears as a part of enveloping curve associated with a family of circles:



The centers of these circles lie on one single (hidden) circle and each circle is tangent to the astroid. An enveloping curve needs not lie on the boundary:



Notice that a slower computer may even serve as a better pedagogical tool, since the attention of the learner has been drawn by the delayed image display.

3 Linkage

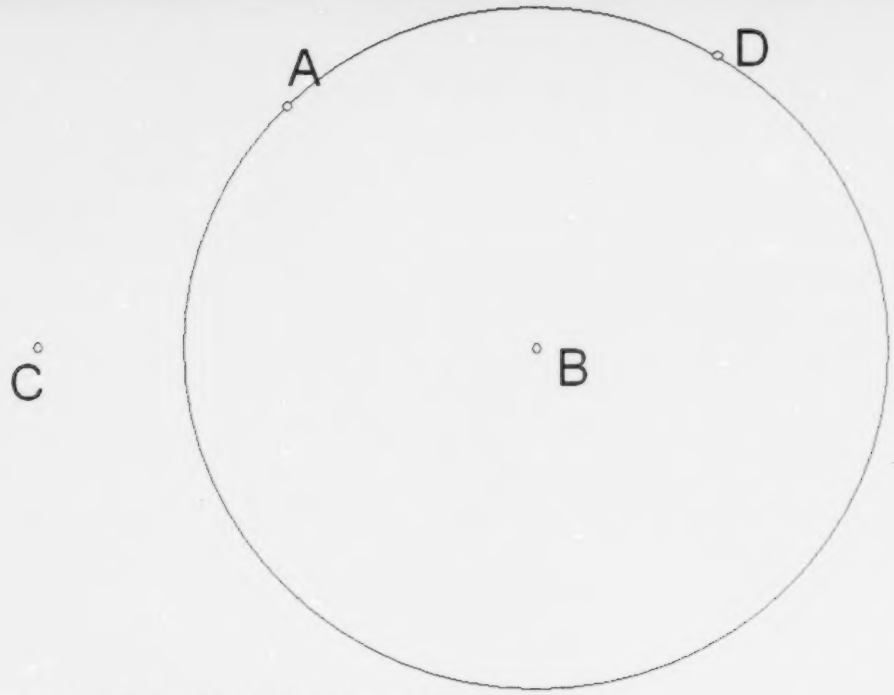
At MIT library one can retrieve a massive book that weighs over 3 kilograms and was published in 1951 in fulfillment of an MS thesis. The title of this dinosaur is called ``Analysis of the Four Bar Linkage".

The four-bar linkage is one of the simplest mechanism and may be regarded as a basic mechanism. The equations which describe the displacement, velocity and acceleration of points on the connecting rod are long and difficult to handle, making mathematical synthesis a very tedious process which few designers will undertake. The publication claims to includes over 7000 displacement paths with the velocity given at 72 equal intervals of drive crank angle along each path and therefore represents about 500,000 solutions of interest to engineers.

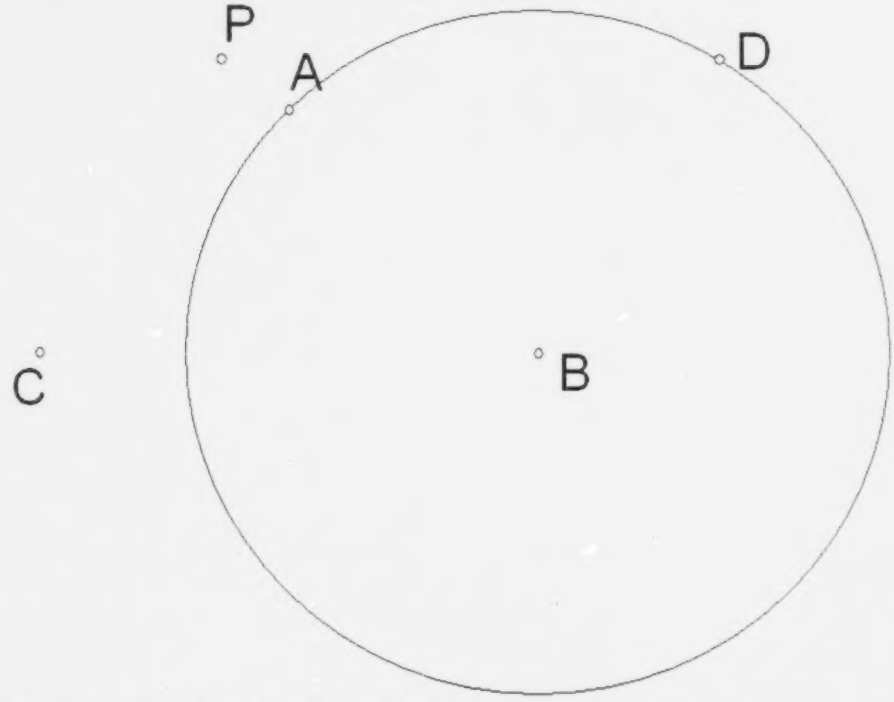
Geometric construction can be adopted to simulate the movement of a linkage. We now show how to construct the Fermat linkage, a particular four-bar linkage used to draw a lemniscate of Bernoulli.

1) Take an equilateral right triangle *ABC*:

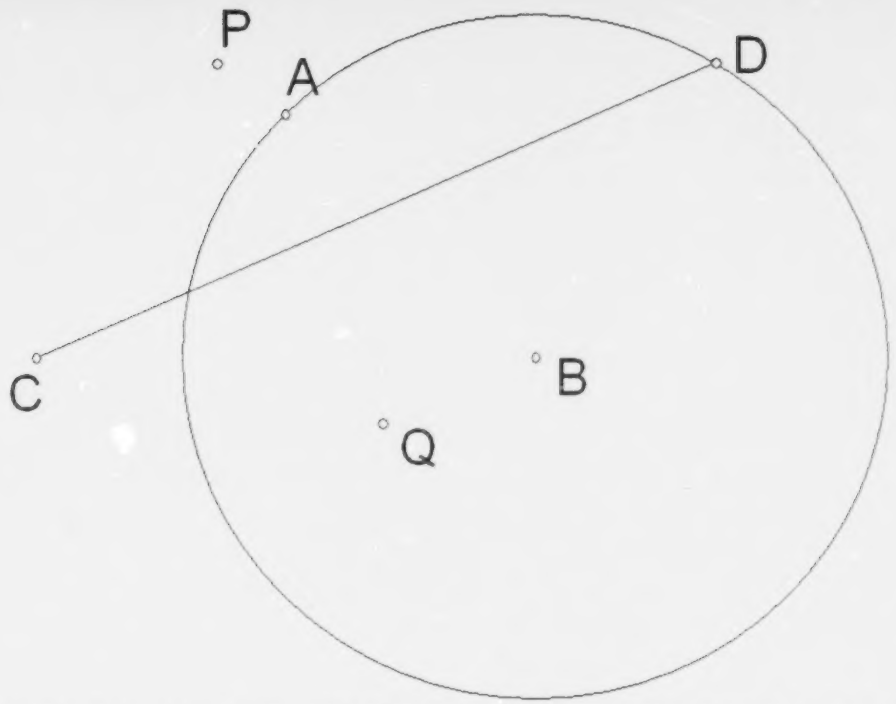
2) With *B* as center, draw a circle passing through *A* and then take a point *D* confined on the circle:



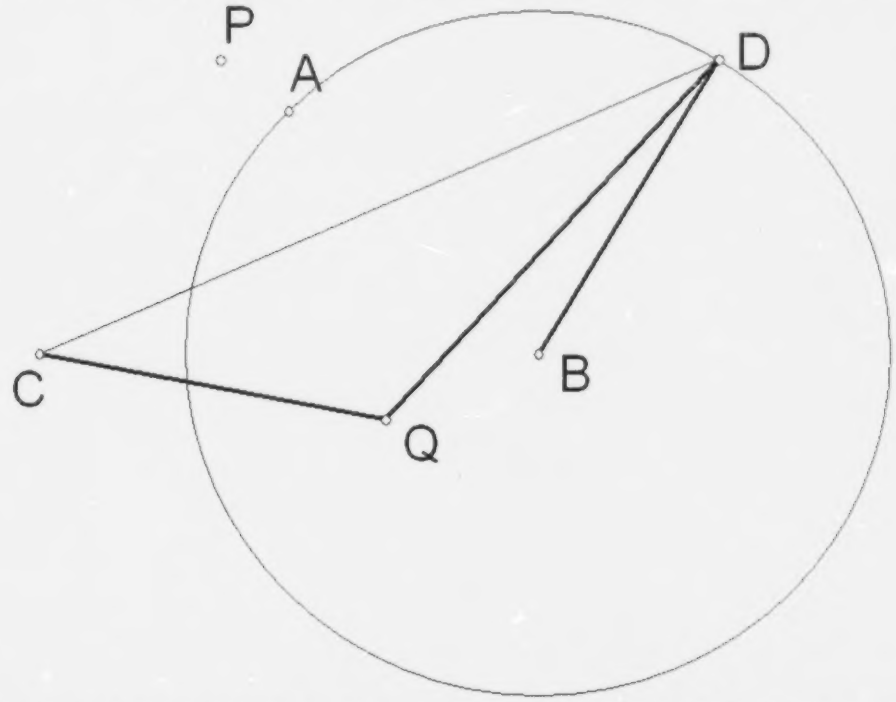
3) Find the point *P* so that *CBDP* is a parallelogram.



4) Construct the reflection *Q* of *P* with respect to the line *CD*:

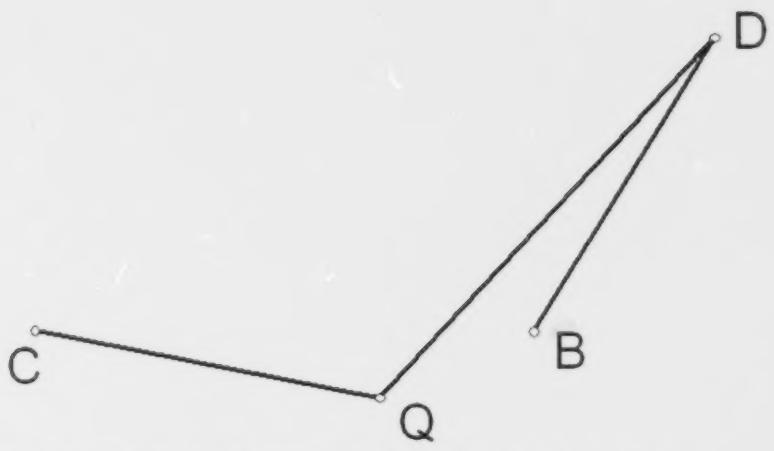


5) Connect the line segments *CQ*,*QD*,*DB*. These are the bars in the linkage:



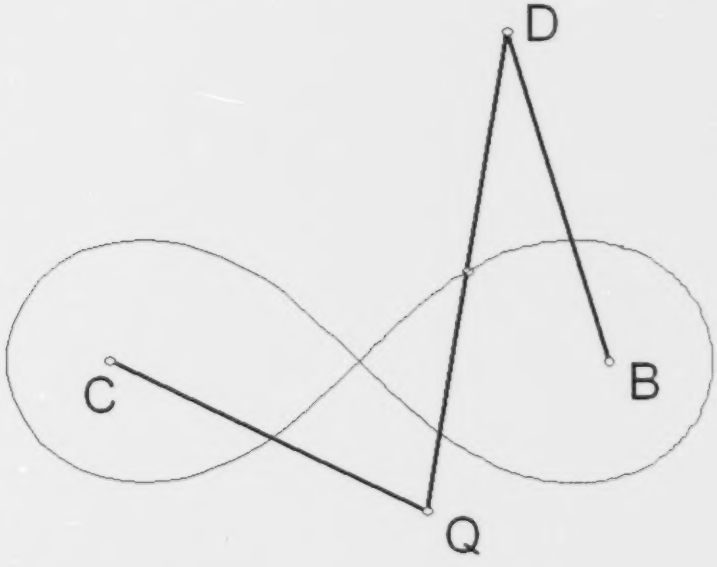
6) Make the point *D* travel along the circle and then hide the circle, the line segment *CD* as well as the points *A* and *P*:

Animate



7) This is the locus of the mid-point of the segment *DQ*, when the Animate button is pressed:

Animate



4 Polynomial Interpolation

Suppose that we are given points $(x_1,y_1),(x_2,y_2),\ldots,(x_n,y_n)$ in the coordinate plane with the x_i distinct. The problem of polynomial interpolation asks for a polynomial function whose graph passes through these points.

Of the many methods of solution to this problem, the ones suggested by Aitken and Neville can be most readily implemented under the dynamical geometric environment. The principle behind constructing the graph of the interpolating polynomial is the following lemma of Aitken:

Lemma. Suppose that the functions f and g satisfy $f(x) = g(x)$ for $x \in T$. If $a \neq b$ then the function

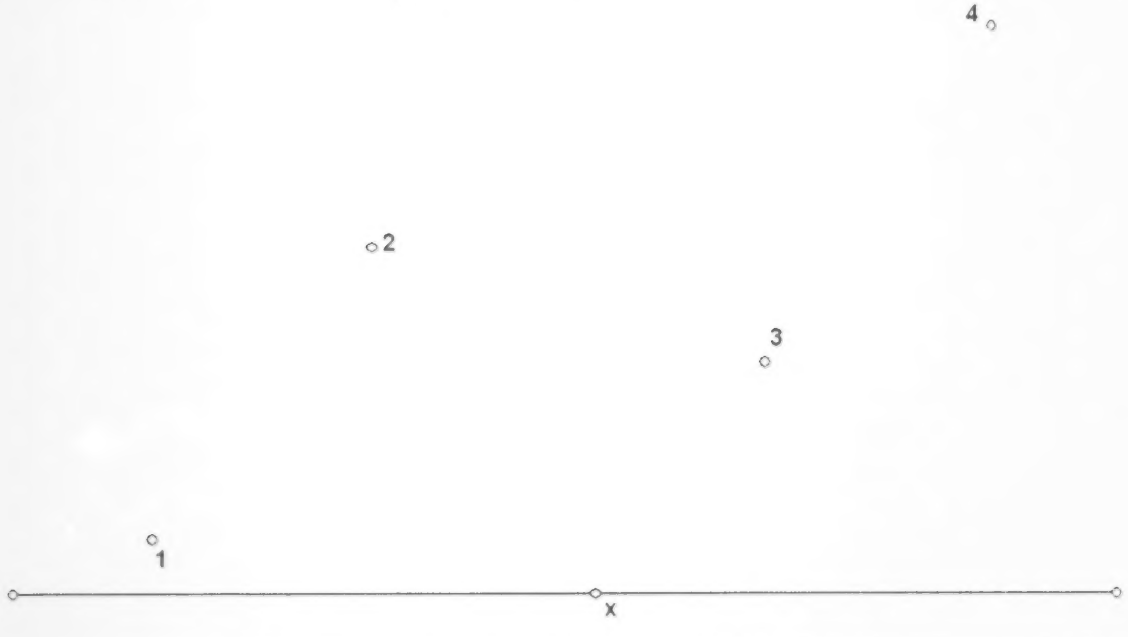
$$h(x) = \frac{(b-x)f(x)+(x-a)g(x)}{b-a}$$

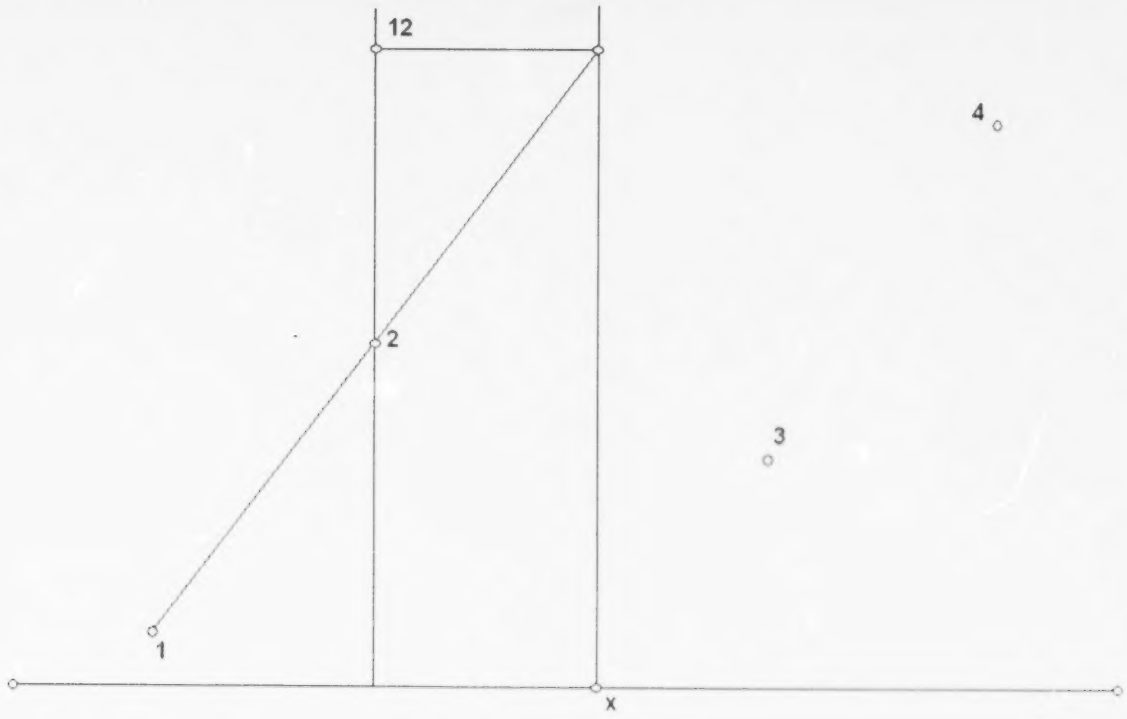
satisfies $f(x) = g(x)$ for $x \in T$, $h(a) = f(a)$, $h(b) = g(b)$.

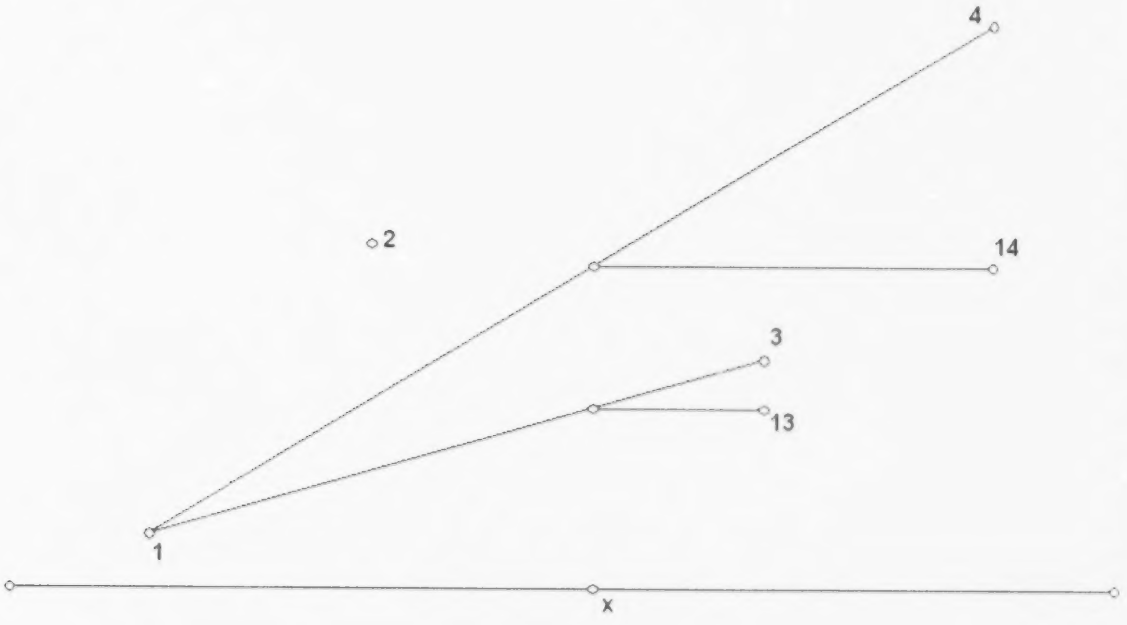
Let $x_1, x_2, \ldots x_n$ be distinct points. Let $y_1, y_2, \ldots y_n$ be given points. For each nonempty subset T of $\{1,2,\ldots,n\}$, let p_T be the (unique) polynomial of degree $\leq |T|-1$ satisfying $p_T(x_j) = y_j$ for $j \in T$. If $|S \setminus T| = 1 = |T \setminus S|$, Aitken's lemma shows us how to produce $p_{S \cup T}$ from p_S and p_T. The required interpolating polynomial $p_{\{1,2,\ldots n\}} \equiv p_{12\ldots n}$ can be computed from this table:

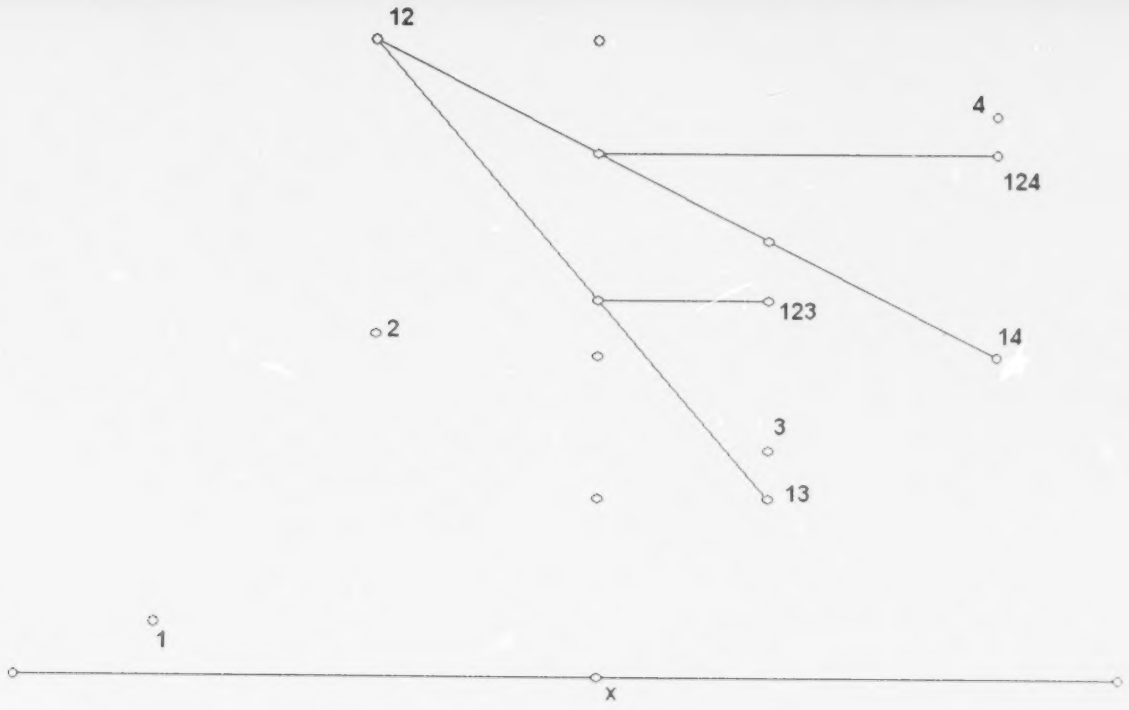
$$\begin{array}{ccccc} p_1 & p_{12} & p_{123} & p_{1234} & p_{12345} \\ p_2 & p_{13} & p_{124} & p_{1235} & \\ p_3 & p_{14} & p_{125} & & \\ p_4 & p_{15} & & & \\ p_5 & & & & \end{array}$$

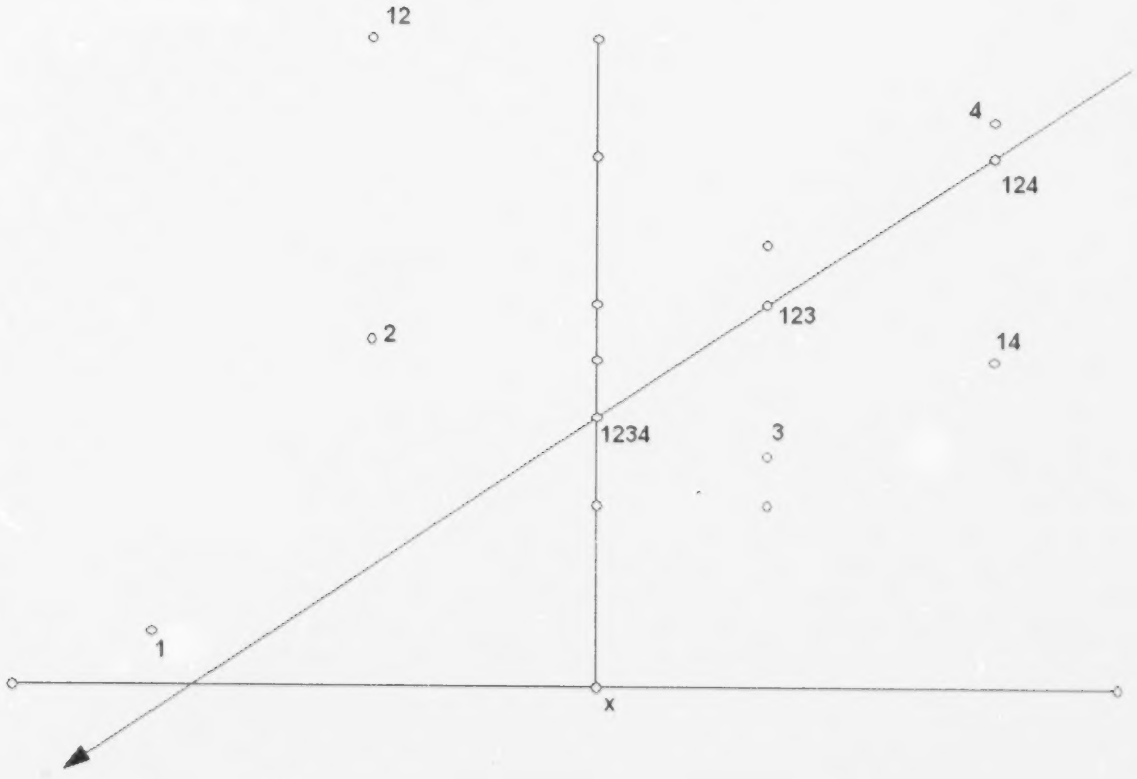
with the obvious initial data $p_j \equiv y_j$ for $j = 1,2,\ldots n$.

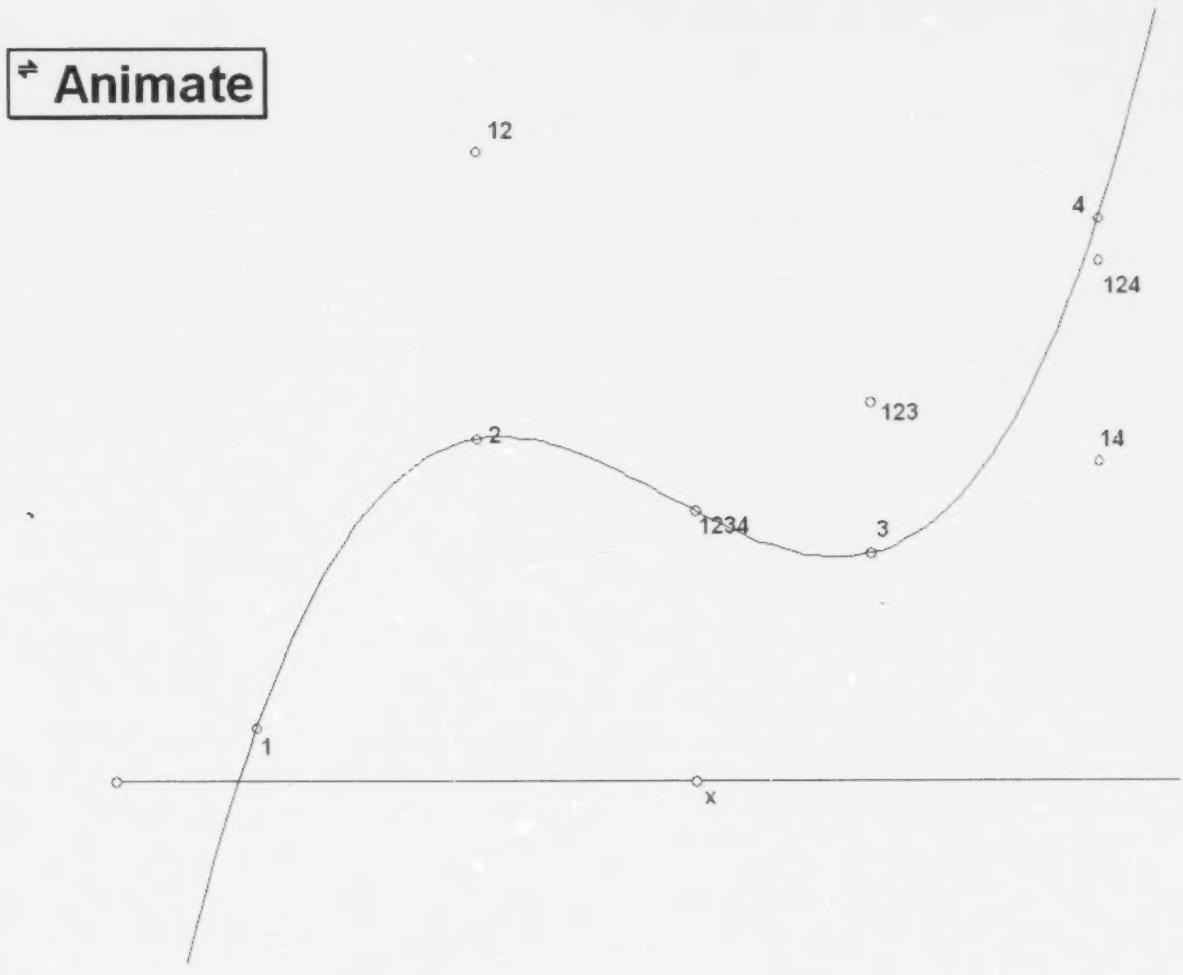



12
4
2
3
1
x


4
2
14
3
13
1
x


12
4
124
123
2
14
3
13
1
x


12
4
124
123
2
14
1234
3
1
x



The preceding procedure may be altered by finding the interpolating polynomial $p_{12\text{键}\cdot n}$ building through this table:

$$p_1\ p_{12}\ p_{123}\ p_{1234}\ p_{12345}$$

$$p_2\ p_{23}\ p_{234}\ p_{2345}$$

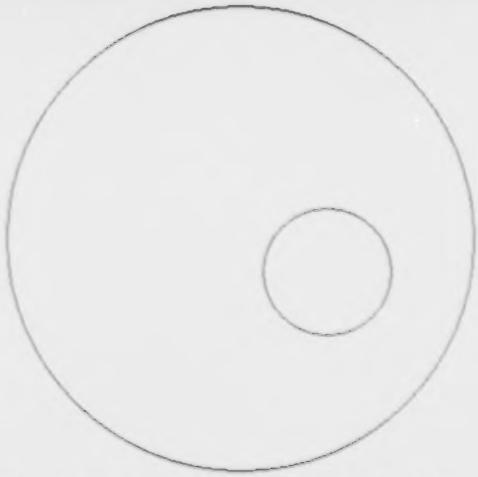
$$p_3\ p_{34}\ p_{345}$$

$$p_4\ p_{45}$$

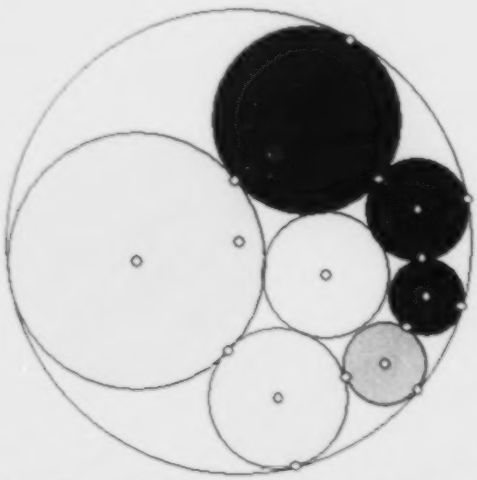
$$p_5$$

5 Inversion

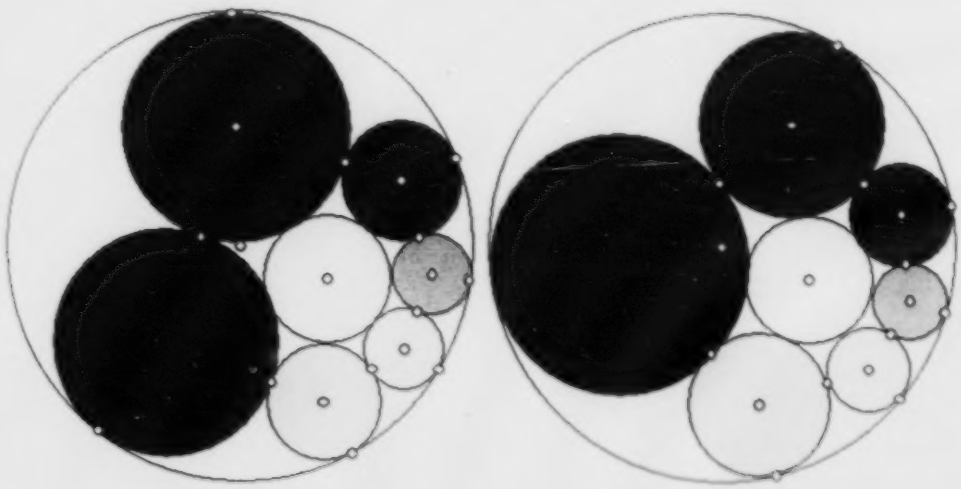
Take two circles, one inside another:

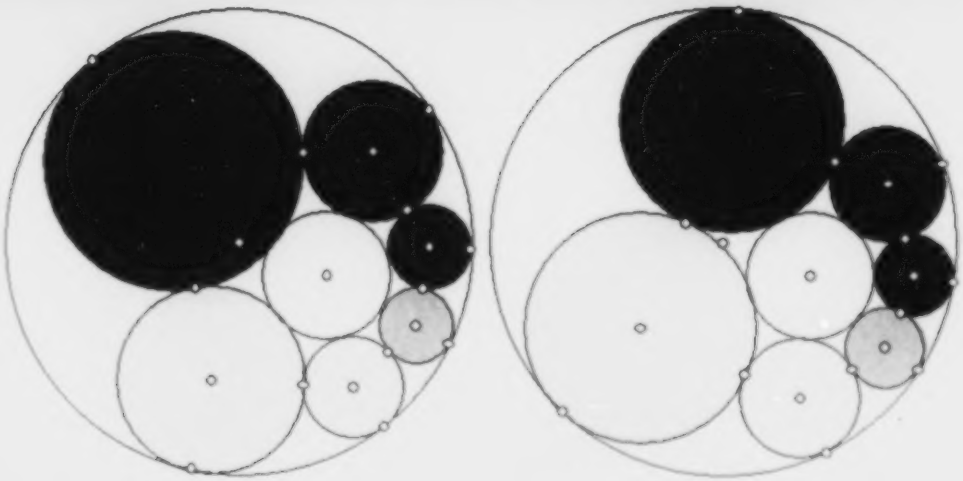


It is not always the case that one can find a series of circles kissing each other so precisely as thus:

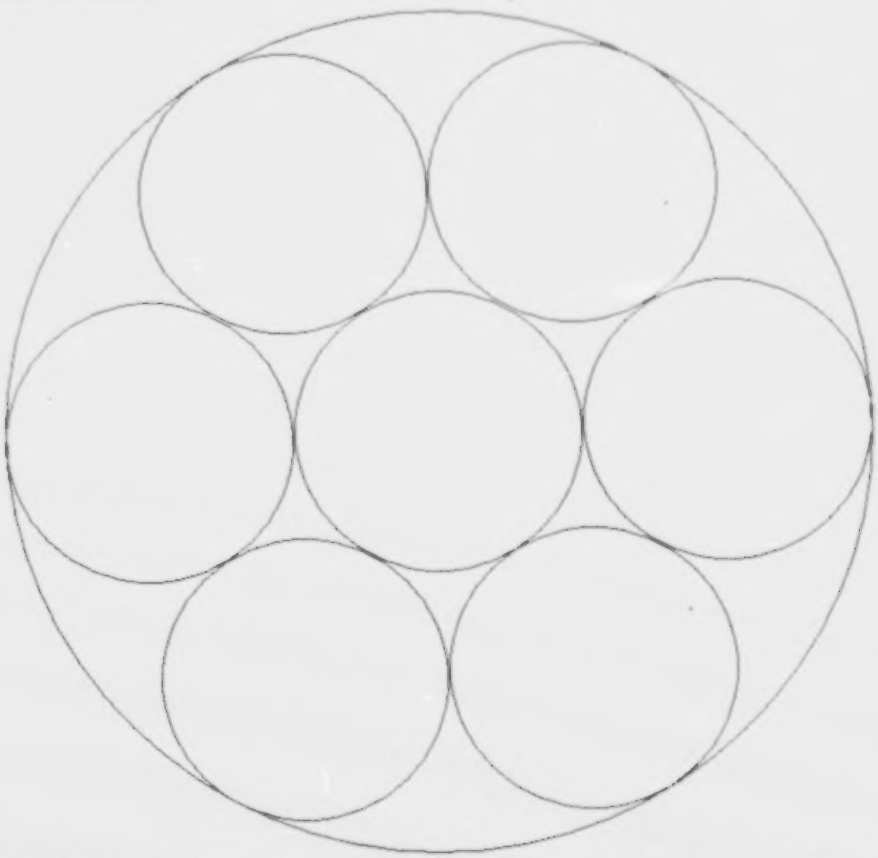


Steiner's porism states that if such situation arises once, it will occur infinitely often:





The principle behind the animation of Steiner porism is inversion. We begin with the particular situation



and then invert the configuration with respect to another circle which is to be hidden. As this symmetric figure rotates, the resulting inverted figure, obeying the law that tangent circles be inverted to tangent circles, achieves the desired result. The method of this demonstration actually guides us to the proof of Steiner's porism! Just invert the two disjoint circles into concentric ones. The configuration of kissing circles close up if and only the inverted figure is symmetric. QED.

6 Epicycloid and Hypocycloid

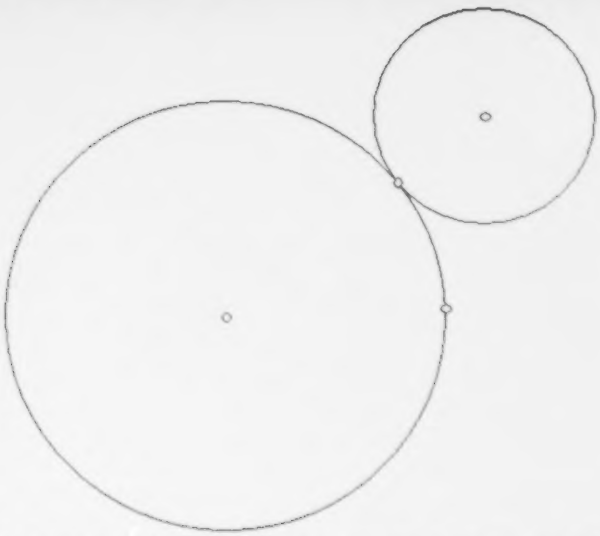
An epicycloid is the curve traced by a point in the circumference of a circle rolling outside a fixed circle. A hypocycloid is the curve traced by a point in the circumference of a circle rolling inside a fixed circle. These curves possess many remarkable properties that fascinate both mathematicians and nonmathematicians alike. It is possible to explore the magic and beauty of these curves through purely geometric constructions. Computer makes it possible for the learner to be exposed to the geometric approach before hooked up to analytical and algebraic methods.

We now show how to generate the epicycloid pictorially:

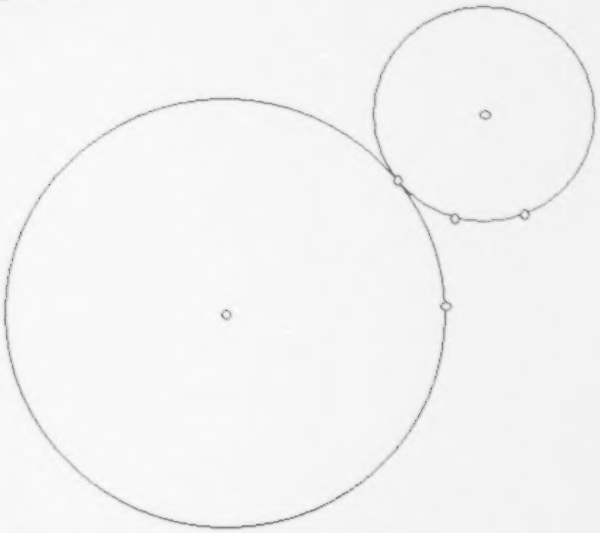
Starting with a circle with ``the point of contact'' attached.

Next, locate the center of the rolling circle.

Draw the rolling circle.

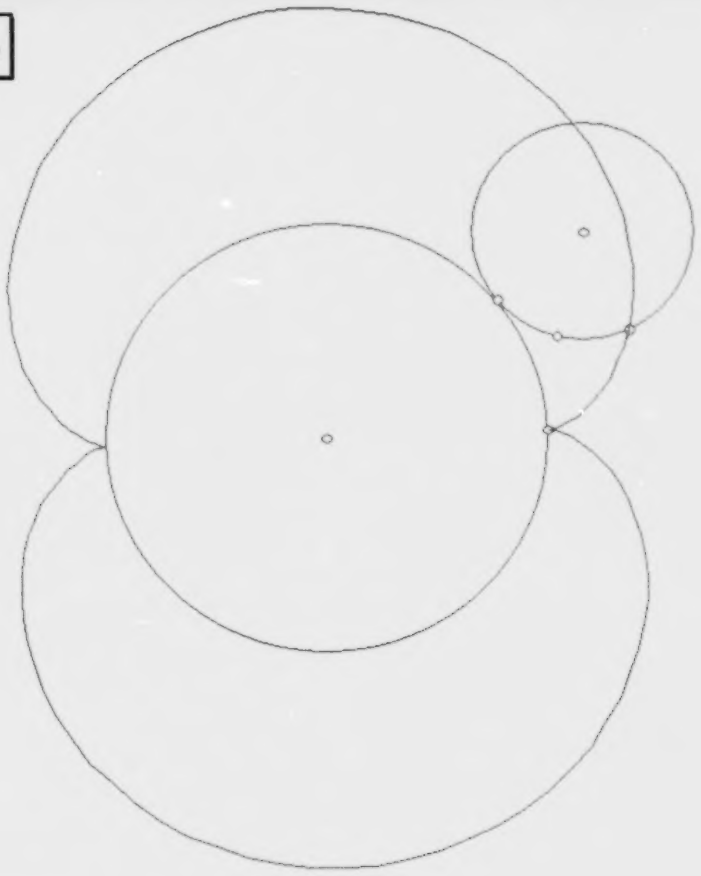


Construct the ``moving point'' by suitably rotating the point of contact w.r.t. the center of the moving circle.

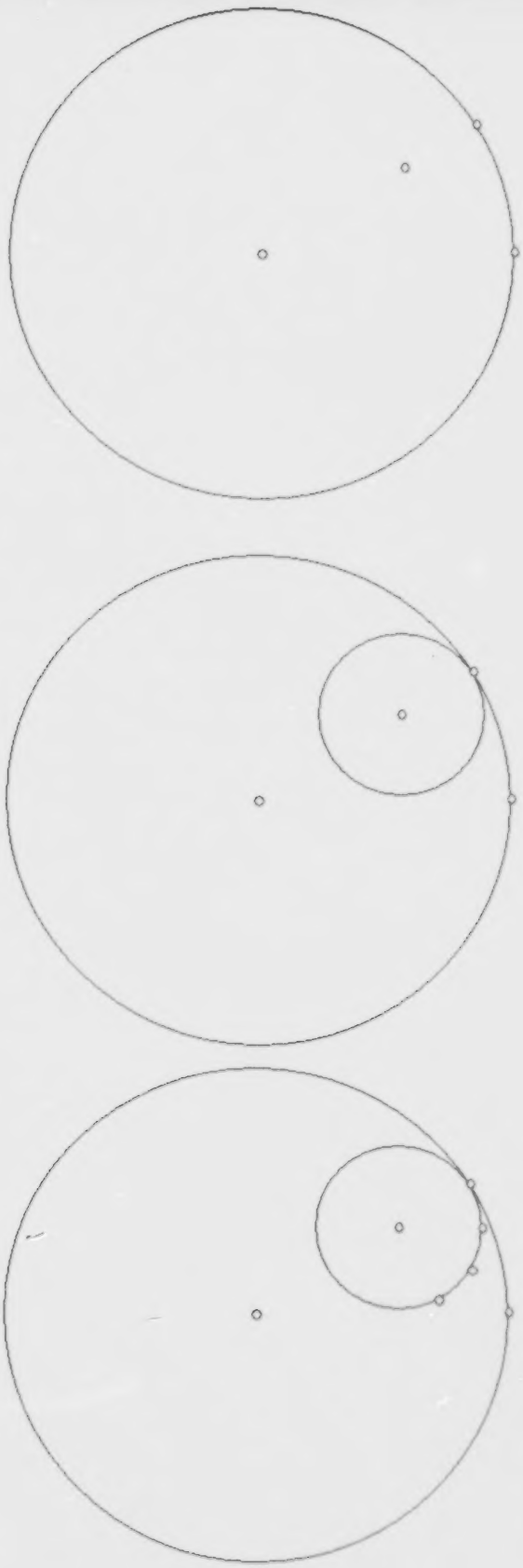


This is the result of pressing the ``Animate'' button:

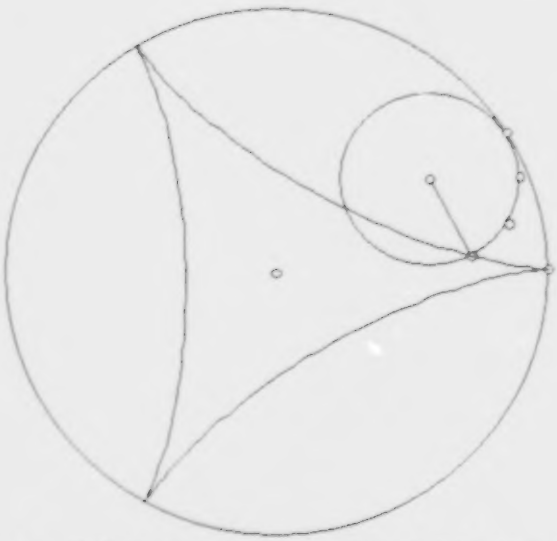
Animate



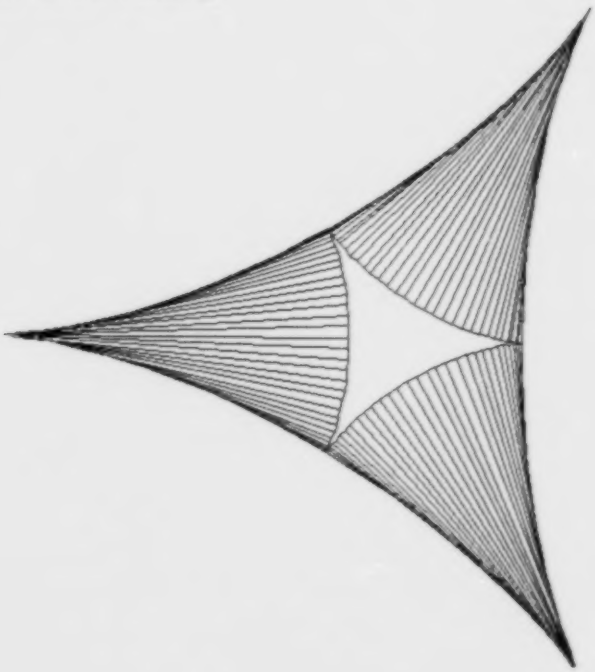
The animation of a hypocycloid is created through a similar process:

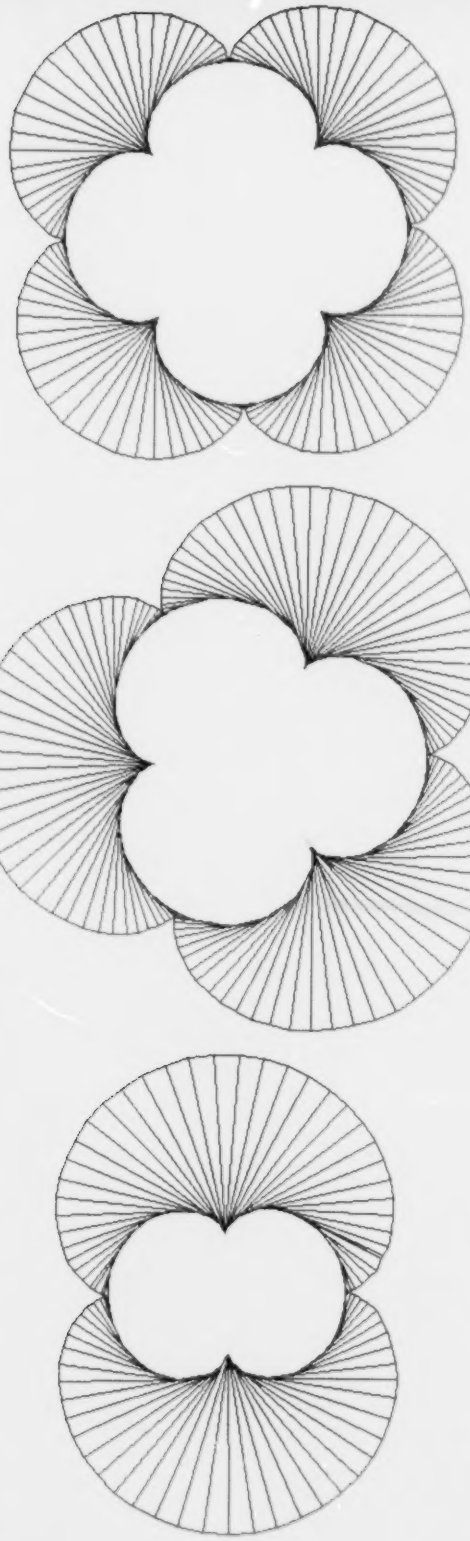


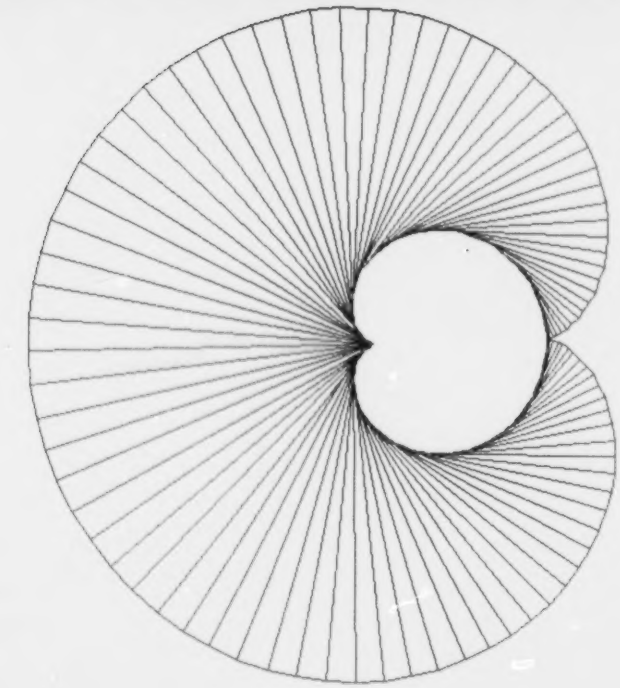
Animate

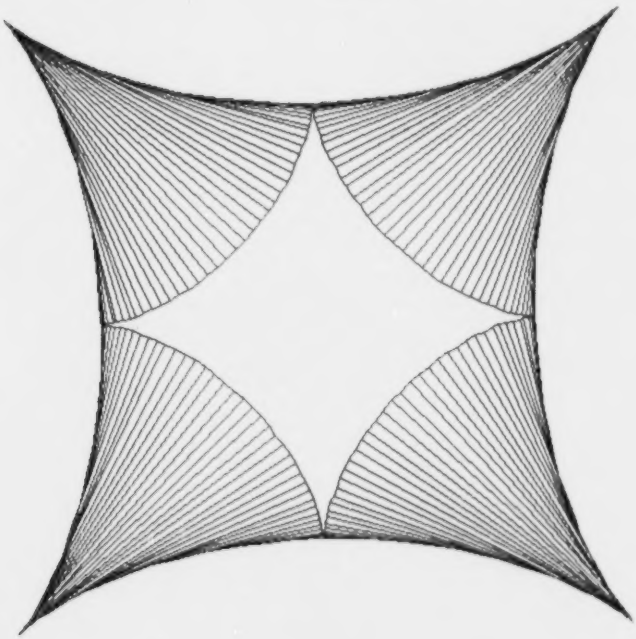


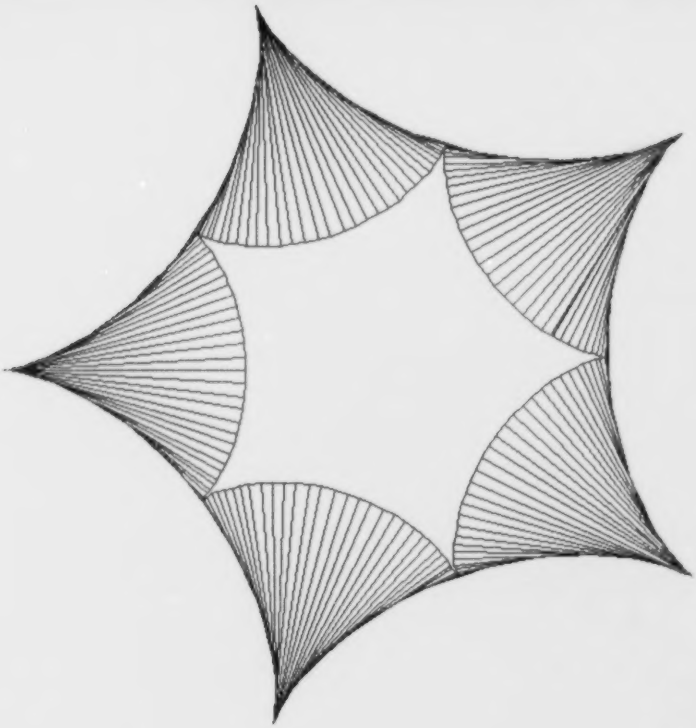
There is a whole host of computer activities related to epicycloids, hypocycloids by drawing the series of line segments joining various points of the curve to the corresponding center of curvature.





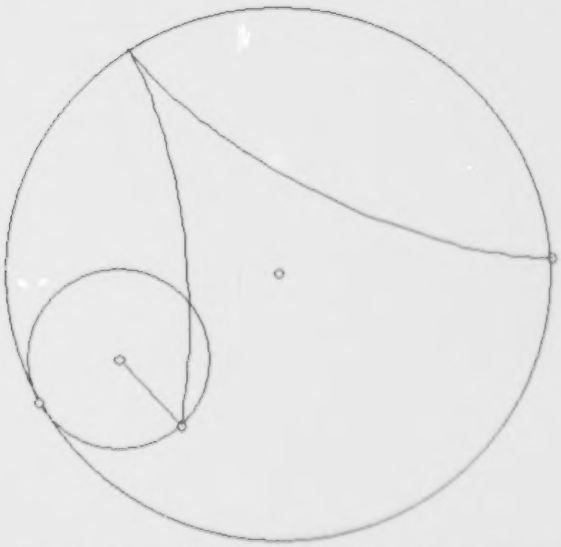






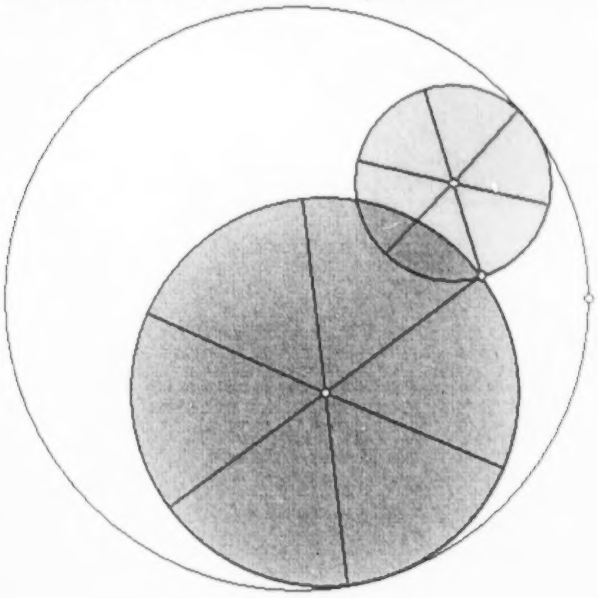
7 Double Generation

Every hypocycloid and every epicycloid can be generated in two ways. Consider the case the hypocycloid traced by a point on the circumference of a rolling circle:



This rolling circle determines another circle rolling in the opposite direction containing the same fixed point on its circumference that traces the hypocycloid. It is difficult to perform a dynamic demonstrate the double generation by means of mechanical devices. With the assistance of computer, however, the

animation of such phenomenon can be vividly shown.



Not convinced just by looking at these pictures? Then you have got to see the demonstration.

References

1. H.S.M. Coxeter, Introduction to Geometry, Wiley, New York, 1961.

2. J. A. Hrones and G. L. Nelson, Analysis of the Four Bar Linkage, Wiley, 1951.

3. E.H. Lockwood, A Book of Curves, Cambridge University Press, 1961.

4. K. Rektorys (Editor), Survey of Applicable Mathematics, Illiffe, London, 1969.

5. J. Todd, Survey of Numerical Analysis, McGraw-Hill, New York, 1962.